SENIOR 'KANGAROO' MATHEMATICAL CHALLENGE

Friday 27th November 2015

Organised by the United Kingdom Mathematics Trust

The Senior Kangaroo paper allows students in the UK to test themselves on questions set for the best school-aged mathematicians from across Europe and beyond.

RULES AND GUIDELINES (to be read before starting):

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. **Use B or HB pencil only** to complete your personal details and record your answers on the machine-readable Answer Sheet provided. **All answers are written using three digits, from 000 to 999.** For example, if you think the answer to a question is 42, write 042 at the top of the answer grid and then code your answer by putting solid black pencil lines through the 0, the 4 and the 2 beneath.

   Please note that the machine that reads your Answer Sheet will only see the solid black lines through the numbers beneath, not the written digits above. You must ensure that you code your answers or you will not receive any marks. There are further instructions and examples on the Answer Sheet.
5. The paper contains 20 questions. Five marks will be awarded for each correct answer. There is no penalty for giving an incorrect answer.
6. The questions on this paper challenge you **to think**, not to guess. Though you will not lose marks for getting answers wrong, you will undoubtedly get more marks, and more satisfaction, by doing a few questions carefully than by guessing lots of answers.

Enquiries about the Senior Kangaroo should be sent to:

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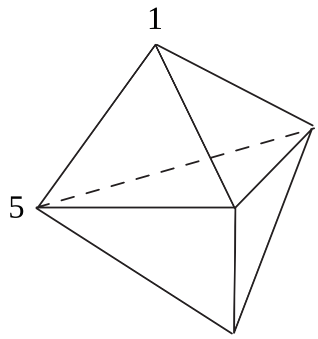
1. In a pile of 200 coins, 2% are gold coins and the rest are silver. Simple Simon removes one silver coin every day until the pile contains 20% gold coins. How many silver coins does Simon remove?

2. The value of the expression $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5}}}$ is $\frac{a}{b}$, where a and b are integers whose only common factor is 1. What is the value of $a + b$?

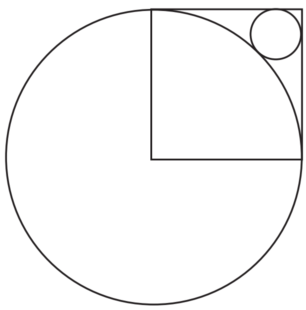
3. The diagram shows a solid with six triangular faces and five vertices. Andrew wants to write an integer at each of the vertices so that the sum of the numbers at the three vertices of each face is the same. He has already written the numbers 1 and 5 as shown.



What is the sum of the other three numbers he will write?

4. A box contains two white socks, three blue socks and four grey socks. Rachel knows that three of the socks have holes in, but does not know what colour these socks are. She takes one sock at a time from the box without looking. How many socks must she take for her to be certain she has a pair of socks of the same colour without holes?

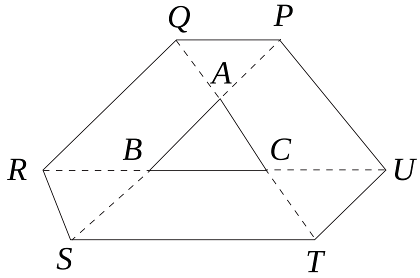
5. The diagram shows two circles and a square with sides of length 10 cm. One vertex of the square is at the centre of the large circle and two sides of the square are tangents to both circles. The small circle touches the large circle. The radius of the small circle is $(a - b\sqrt{2})$ cm.



What is the value of $a + b$?

6. The median of a set of five positive integers is one more than the mode and one less than the mean. What is the largest possible value of the range of the five integers?

7. The diagram shows a triangle ABC with area 12 cm^2. The sides of the triangle are extended to points P, Q, R, S, T and U as shown so that $PA = AB = BS$, $QA = AC = CT$ and $RB = BC = CU$.



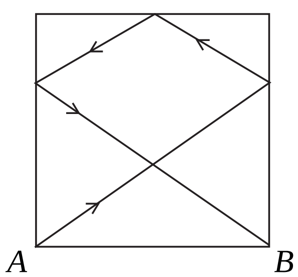
What is the area (in cm^2) of hexagon $PQRSTU$?

8. A mob of 2015 kangaroos contains only red and grey kangaroos. One grey kangaroo is taller than exactly one red kangaroo, one grey kangaroo is taller than exactly three red kangaroos, one grey kangaroo is taller than exactly five red kangaroos and so on with each successive grey kangaroo being taller than exactly two more red kangaroos than the previous grey kangaroo. The final grey kangaroo is taller than all the red kangaroos. How many grey kangaroos are in the mob?

9. A large rectangle is divided into four identical smaller rectangles by slicing parallel to one of its sides. The perimeter of the large rectangle is 18 metres more than the perimeter of each of the smaller rectangles. The area of the large rectangle is 18 m^2 more than the area of each of the smaller rectangles. What is the perimeter in metres of the large rectangle?

10. Katherine and James are jogging in the same direction around a pond. They start at the same time and from the same place and each jogs at a constant speed. Katherine, the faster jogger, takes 3 minutes to complete one lap and first overtakes James 8 minutes after starting. How many seconds does it take James to complete one lap?

11. A ball is propelled from corner A of a *square* snooker table of side 2 metres. After bouncing off three cushions as shown, the ball goes into a pocket at B. The total distance travelled by the ball is $\sqrt{k}$ metres. What is the value of k?



(Note that when the ball bounces off a cushion, the angle its path makes with the cushion as it approaches the point of impact is equal to the angle its path makes with the cushion as it moves away from the point of impact as shown in the diagram below.)

12. Chris planned a 210 km bike ride. However, he rode 5 km/h faster than he planned and finished his ride 1 hour earlier than he planned. His average speed for the ride was x km/h. What is the value of x?

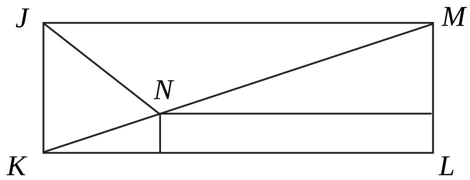
13. Twenty-five people who always tell the truth or always lie are standing in a queue. The man at the front of the queue says that everyone behind him always lies. Everyone else says that the person immediately in front of them always lies. How many people in the queue always lie?

14. Four problems were attempted by 100 contestants in a Mathematics competition. The first problem was solved by 90 contestants, the second by 85 contestants, the third by 80 contestants and the fourth by 75 contestants. What is the smallest possible number of contestants who solved all four problems?

15. The 5-digit number '$XX4XY$' is exactly divisible by 165. What is the value of $X + Y$?

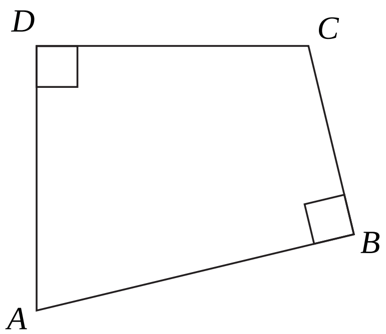
16. How many 10-digit numbers are there whose digits are all 1, 2 or 3 and in which adjacent digits differ by 1?

17. In rectangle $JKLM$, the bisector of angle KJM cuts the diagonal KM at point N as shown. The distances between N and sides LM and KL are 8 cm and 1 cm respectively. The length of KL is $(a + \sqrt{b})$ cm. What is the value of $a + b$?



18. Numbers a, b and c are such that $\dfrac{a}{b + c} = \dfrac{b}{c + a} = \dfrac{c}{a + b} = k$.
How many possible values of k are there?

19. In quadrilateral $ABCD$, $\angle ABC = \angle ADC = 90°$, $AD = DC$ and $AB + BC = 20$ cm.



What is the area in cm^2 of quadrilateral $ABCD$?

20. The number $N = 3^{16} - 1$ has a divisor of 193. It also has some divisors between 75 and 85 inclusive. What is the sum of these divisors?